Calculus II - Applet Lab 1  
Area Accumulation Functions and FTC - II

Begin this lab by revisiting Example 4. There you will find that the function \( f(x) \) under which area is being accumulated is given by:

\[
    f(x) = \begin{cases} 
        x & x \leq 1 \\
        -x + 2 & x > 1 
    \end{cases}
\]

With \( C = 0 \), the applet produces a plot of \( F(x) = \int_0^x f(t) \, dt \), an antiderivative of \( f(x) \), by accumulating area under the graph of \( f \) using \( a = 0 \) as the specific starting location of the area accumulation. In particular, \( F(0) = 0 \) for this antiderivative.

**Question (1):** Find the functional formulation for \( F(x) \). Your answer should be a piecewise defined function similar to the one given for \( f(x) \) above. Use precise mathematical language to clearly explain how you arrived at your answer.

Since \( F(x) \) is an antiderivative of \( f(x) \), you know (See Theorem 1, Section 4.9) that any other antiderivative \( G(x) \) of \( f(x) \) will have the form \( G(x) = F(x) + C \). Geometrically, \( C \) representing a vertical translation of the graph of \( F(x) \). Vary \( C \) in the applet to see this.

**Question (2):** Carefully explain the connection between the shapes of \( F(x) \) and \( G(x) \), the derivatives of \( F(x) \) and \( G(x) \), the vertical translation \( C \), and values of \( f(x) \).

Finally, since \( f(x) \) is continuous on \((-\infty, \infty)\), the specific location \( x = a \) where area accumulation is started can be any real number leading to an antiderivative of \( f(x) \) of the form \( F_a(x) = \int_a^x f(t) \, dt \).

**Question (3):** Change the value of \( C \) in the applet to \(-1\). Now, find 2 distinct values for \( a \) that lead to \( F_a(0) = -1 \). For each value that you find, show that \( F_a(0) = \int_a^0 f(t) \, dt = -1 \) by directly determining the value of the definite integral.